Pearson Edexcel

Mark Scheme (Results)

Summer 2018

Pearson Edexcel GCE A Level Mathematics
Core Mathematics C3 (6665)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = …

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = …

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0,\ \ q \neq 0$, leading to x = …

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
1.(a)	$y=2x(3x-1)^5 \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}=2(3x-1)^5+30x(3x-1)^4$	M1A1
	$\Rightarrow \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)=2(3x-1)^4\{(3x-1)+15x\}=2(3x-1)^4(18x-1)$	M1A1
		(4)
(b)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}\leqslant 0 \Rightarrow 2(3x-1)^4(18x-1)\leqslant 0 \Rightarrow x\leqslant \dfrac{1}{18} \quad x=\dfrac{1}{3}$	B1ft, B1
		(2)
		(6 marks)

This may be marked as one complete question

(a)

M1: Uses the product rule $vu'+uv'$ with $u=2x$ and $v=(3x-1)^5$ or vice versa to achieve an expression of the form $A(3x-1)^5+Bx(3x-1)^4$, $A,B>0$

Condone slips on the $(3x-1)$ and $2x$ terms but misreads on the question must be of equivalent difficulty. If in doubt use review.

Eg: $y=2x(3x+1)^5 \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}=2(3x+1)^5+30x(3x+1)^4$ can potentially score 1010 in (a) and 11 in (b)

Eg: $y=2x(3x+1)^{15} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}=2(3x+1)^{15}+90x(3x+1)^{14}$ can potentially score 1010 in (a) and 11 in (b)

Eg: $y=2(3x+1)^5 \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}=30(3x+1)^4$ is 0000 even if attempted using the product rule (as it is easier)

A1: A correct un-simplified expression. You may never see the lhs which is fine for all marks.

M1: Scored for taking a common factor of $(3x-1)^4$ out of $A(3x-1)^5 \pm Bx^n(3x-1)^4$ where $n=1$ or 2,to reach a form $(3x-1)^4\{........\}$ You may condone one slip in the $\{........\}$

Alternatively they take out a common factor of $2(3x-1)^4$ which can be scored in the same way

Example of one slip $2(3x-1)^5+30x(3x-1)^4=(3x-1)^4\{(3x-1)+30x\}$

If a different form is reached, see examples above, it is for equivalent work.

A1: Achieves a fully factorised simplified form $2(3x-1)^4(18x-1)$ which may be awarded in (b)

(b)

B1ft: For a final answer of either $x\leqslant\dfrac{1}{18}$ or $x=\dfrac{1}{3}$ Condone $x\leqslant\dfrac{2}{36}$ $x\le 0.0\dot{5}$ $x=0.\dot{3}$

Do not allow $x=\dfrac{1}{3}$ if followed by $x\leqslant\dfrac{1}{3}$ Follow through on a linear factor of $(Ax+B)\leqslant 0 \Rightarrow x...$ where $A,B\neq 0$. Watch for negative A's where the inequality would reverse.

It may be awarded within an equality such as $\dfrac{1}{3}\leqslant \; x\leqslant\dfrac{1}{18}$

B1: **For a final answer** of $x\leqslant\dfrac{1}{18}$ oe (and) $x=\dfrac{1}{3}$ oe with no other solutions. Ignore any references to and/or here. Misreads can score these marks

Question Number	Scheme	Marks
2(a)	$4x^2-25 \to (2x+5)(2x-5)$	B1
	$\frac{6}{2x+5}+\frac{2}{2x-5}+\frac{60}{4x^2-25}=\frac{6(2x-5)+2(2x+5)+60}{(2x+5)(2x-5)}$	M1
	$=\frac{16x+40}{(2x+5)(2x-5)}$	A1
	$=\frac{8(2x+5)}{(2x+5)(2x-5)}=\frac{8}{(2x-5)}$	A1
		(4)
(b)	$\mathrm{f}(x)=\frac{8}{2x-5} \Rightarrow y=\frac{8}{2x-5} \Rightarrow 2xy-5y=8 \Rightarrow x=\frac{8+5y}{2y}$	M1
	$\Rightarrow \mathrm{f}^{-1}(x)=\frac{8+5x}{2x}$ oe	A1
	$0<x<\frac{8}{3}$	B1ft
		(3)
		(7 marks)

Alternative solutions to part (a)

2(a) ALT I	$4x^2-25=(2x+5)(2x-5)$	B1
	$\frac{6}{2x+5}+\frac{2}{2x-5}=\frac{16x-20}{4x^2-25}$	
	$\frac{6}{2x+5}+\frac{2}{2x-5}+\frac{60}{4x^2-25}=\frac{16x-20+60}{4x^2-25}$	M1
	$=\frac{16x+40}{4x^2-25}$	A1
	$=\frac{8(2x+5)}{(2x+5)(2x-5)}=\frac{8}{2x-5}$	A1

2(a) ALT II	$4x^2-25=(2x+5)(2x-5)$	B1
	$\frac{60}{4x^2-25}=\frac{-6}{2x+5}+\frac{6}{2x-5}$	M1
	$\frac{6}{2x+5}+\frac{2}{2x-5}+\frac{60}{4x^2-25}=\frac{6}{2x+5}+\frac{2}{2x-5}+\frac{-6}{2x+5}+\frac{6}{2x-5}$	A1
	$=\frac{8}{(2x-5)}$	A1

(a)

B1: For **factorising** $4x^2-25\rightarrow(2x+5)(2x-5)$ This can occur anywhere in the solution.

Note that it is possible to score this mark for expanding $(2x+5)(2x-5)\rightarrow 4x^2-25$ and then cancelling by $4x^2-25$. Both processes are required by this route. It can be implied if you see the correct intermediate form. (See A1)

M1: For combining the three fractions with a common denominator. The denominator must be correct and at least one numerator must have been adapted correctly . Accept as separate fractions. Condone missing brackets.

Accept $\frac{6}{2x+5}+\frac{2}{2x-5}+\frac{60}{4x^2-25}=\frac{6(2x-5)\left(4x^2-25\right)+2(2x+5)\left(4x^2-25\right)+60(2x+5)(2x-5)}{(2x+5)(2x-5)\left(4x^2-25\right)}$

Condone $\frac{6}{2x+5}+\frac{2}{2x-5}+\frac{60}{4x^2-25}=\frac{6(2x-5)+2+60}{(2x+5)(2x-5)}$ correct denominator, one numerator adapted correctly

Alternatively uses partial fractions $\frac{60}{4x^2-25}=\frac{A}{2x+5}+\frac{B}{2x-5}$ leading to values for *A* and *B*

A1: A correct intermediate form of $\frac{\text{simplified linear}}{\text{quadratic}}$ most likely to be $\frac{16x+40}{(2x+5)(2x-5)}$

Sometimes the candidate may write out the simplified numerator separately. In cases like this, you can award this A mark without explicitly seeing the fraction as long as a correct denominator is seen.

Using the partial fraction method, it is for $\frac{6}{2x+5}+\frac{2}{2x-5}+\frac{60}{4x^2-25}=\frac{6}{2x+5}+\frac{2}{2x-5}+\frac{-6}{2x+5}+\frac{6}{2x-5}$

A1: Further factorises and cancels (all of which may be implied) to reach the answer $\frac{8}{2x-5}$

This is not a given answer so condone slips in bracketing etc.

(b)

M1: Attempts to change the subject of the formula for a function of the form $y=\frac{A}{Bx+C}$

Condone attempts on an equivalent made up equation for candidates who don't progress in part (a). As a minimum expect to see multiplication by $(Bx+C)$ leading to x (or a replaced y) =

Alternatively award for 'inverting' Eg. $y=\frac{A}{Bx+C}$ to $\frac{Bx+C}{A}=\frac{1}{y}$ leading to x (or a replaced y) =

A1: $\mathrm{f}^{-1}(x)=\frac{8+5x}{2x}$ or $y=\frac{8+5x}{2x}$ or equivalent. Accept $y=\frac{4}{x}+\frac{5}{2}$ Condone $\mathrm{F}^{-1}(x)=\frac{8+5x}{2x}$

Condone $y=\frac{1}{2}\left(\frac{8}{x}+5\right)$ and $y=\frac{8}{2x}+\frac{5}{2}$ **BUT NOT** $y=\frac{\frac{8}{x}+5}{2}$ (fractions within fractions)

You may isw after a correct answer.

B1ft: $0<x<\frac{8}{3}$ or alternative forms such as $0<\textit{Domain}<\frac{8}{3}$ Domain $=\left(0,\frac{8}{3}\right)$ or $\frac{8}{3}>x>0$

Do not accept $0<y<\frac{8}{3}$ or $0<\mathrm{f}^{-1}(x)<\frac{8}{3}$

Follow through on their $y=\frac{A}{Bx+C}$ so accept $0<x<\frac{A}{4B+C}$

Question Number	Scheme	Marks
3(a)	$A=1500$	B1
		(1)
(b)	Sub $t=2, V=13500 \Rightarrow 16000\mathrm{e}^{-2k}=12000$	M1
	$\Rightarrow e^{-2k}=\frac{3}{4}\quad 0.75\quad$ oe	A1
	$\Rightarrow k=-\frac{1}{2}\ln\frac{3}{4},=\ln\sqrt{\frac{4}{3}}=\ln\left(\frac{2}{\sqrt{3}}\right)$	dM1, A1*
		(4)
(c)	Sub $6000=16000\mathrm{e}^{-\ln\left(\frac{2}{\sqrt{3}}\right)T}+'1500' \Rightarrow \mathrm{e}^{-\ln\left(\frac{2}{\sqrt{3}}\right)T}=C$	M1
	$\Rightarrow \mathrm{e}^{-\ln\left(\frac{2}{\sqrt{3}}\right)T}=\frac{45}{160}=\ 0.28125$	A1
	$\Rightarrow T=-\frac{\ln\left(\frac{45}{160}\right)}{\ln\left(\frac{2}{\sqrt{3}}\right)}=8.82$	M1 A1
		(4)
		(9 marks)
Alt (b)	Sub $t=2, V=13500 \Rightarrow 13500{=}16000\mathrm{e}^{-2k}+'1500' \Rightarrow 1600e^{-2k}=1200$	M1
	$\Rightarrow \ln 1600-2k=\ln 1200$	A1
	$\Rightarrow k=-\frac{1}{2}\ln\frac{1200}{1600},=\ln\sqrt{\frac{4}{3}}=\ln\left(\frac{2}{\sqrt{3}}\right)$	dM A1*
		(4)

You may mark parts (a) and (b) together

(a)

B1: Sight of $A=1500$

(b)

M1: Substitutes $t=2, V=13500 \Rightarrow 13500{=}16000\mathrm{e}^{-2k}+'\mathit{their}\,1500'$ and proceeds to $P\mathrm{e}^{-2k}=...$ or $Q\mathrm{e}^{2k}=...$ Condone slips, for example, V may be 1350. It is for an **attempt** to make $\mathrm{e}^{\pm 2k}$ the subject.

A1: $e^{-2k}=\frac{3}{4}\quad 0.75\quad$ or $e^{2k}=\frac{4}{3}\quad \left(1.\dot{3}\right)$ oe

dM1: For taking ln's and proceeding to k =... For example $k=-\frac{1}{2}\ln\frac{3}{4}$ oe

May be implied by the correct decimal answer awrt 0.144 . This mark cannot be awarded from impossible to solve equations, that is ones of the type $\Rightarrow e^{\pm 2k}=c,\quad c\leqslant 0$

A1*: cso $k=\ln\left(\frac{2}{\sqrt{3}}\right)$ (brackets not required) **with a correct intermediate line** of either

$$\frac{1}{2}\ln\frac{4}{3},\ \frac{1}{2}\ln 4-\frac{1}{2}\ln 3,\ \ln\sqrt{\frac{4}{3}}\ \text{ or }\ \ln\left(\frac{3}{4}\right)^{-\frac{1}{2}}$$

Note: $e^{-2k}=\frac{3}{4}\Rightarrow e^{2k}=\frac{4}{3}\Rightarrow e^{k}=\frac{2}{\sqrt{3}}$ are perfectly acceptable steps

See scheme for alternative method when ln's are taken before e^{-2k} is made the subject.

It is also possible to substitute $k=\ln\left(\frac{2}{\sqrt{3}}\right)$ into $13500=16000e^{-k\times 2}+1500$ and show that 12000 =12000 or similar. This is fine as long as a minimal conclusion (eg ✓)is given for the A1*.

(c)

M1: Sub $V=6000\Rightarrow 6000=16000e^{\pm kT}+\text{'their 1500'}$ and proceeds to $e^{\pm kT}=c,\quad c>0$

Allow candidates to write $k=$ awrt 0.144 or leave as '*k*'. Condone slips on *k*. Eg $k=2\ln\left(\frac{2}{\sqrt{3}}\right)$

Allow this when the = sign is replaced by any inequality.

If the candidate attempts to simplify the exponential function score for $\left(\frac{2}{\sqrt{3}}\right)^{\pm T}=c,\quad c>0$

A1: $e^{-\ln\left(\frac{2}{\sqrt{3}}\right)T}=\frac{45}{160}=0.28125$, $e^{-kT}=\frac{45}{160}$ or $\left(\frac{2}{\sqrt{3}}\right)^{-T}=\frac{45}{160}$ Condone inequalities for =

Allow solutions from rounded values (3sf). Eg. $e^{-0.144T}=0.281$

M1: Correct order of operations using ln's and division leading to a value of T. It is implied by awrt 8.8

$\left(\frac{2}{\sqrt{3}}\right)^{-T}=\frac{45}{160}\Rightarrow -T=\log_{\frac{2}{\sqrt{3}}}\frac{45}{160}$ is equivalent work for this M mark.

A1: cso 8.82 only following correct work. Note that this is not awrt

Allow a solution using an inequality as long as it arrives at the solution 8.82.

There may be solutions using trial and improvement. Score (in this order) as follows

M1: Trial at value of $V=16000e^{-0.144t}+1500$ (oe) at either t = 8 or t = 9 and shows evidence $V_{t=8}=awrt\ 6500$ $V_{t=9}=awrt\ 5900$ This may be implied by the subsequent M1

M1: Trial at value of $V=16000e^{-0.144t}+1500$ (oe) at either t = 8.81 or t = 8.82 and shows evidence. (See below for answers. Allow to 2sf)

A1: Correct answers for *V* at **both** t = 8.81 **and** t = 8.82 $V_{t=8.81}=awrt\ 6006$ $V_{t=8.82}=awrt\ 5999$

A1: Correctly deduces 8.82 with all evidence.

Hence candidates who **just** write down 8.82 will score 1, 1, 0, 0

Question Number	Scheme	Marks
4.(a)	$\frac{dy}{dx}=-2e^{-2x}+2x$	M1A1
	At $x=0$ $\frac{dy}{dx}=-2\Rightarrow\frac{dx}{dy}=\frac{1}{2}$	M1
	Equation of normal is $y-(-2)=\frac{1}{2}(x-0)\Rightarrow y=\frac{1}{2}x-2$	M1 A1
		(5)
(b)	$y=e^{-2x}+x^2-3$ meets $y=\frac{1}{2}x-2$ when $e^{-2x}+x^2-3="\frac{1}{2}x-2"$	
	$x^2=1+\frac{1}{2}x-e^{-2x}$	M1
	$x=\sqrt{1+\frac{1}{2}x-e^{-2x}}$ *	A1*
		(2)
(c)	$x_2=\sqrt{1+0.5-e^{-2}}$	M1
	$x_2=1.168,\ x_3=1.220$	A1
		(2)
		(9 marks)

(a)

M1: Attempts to differentiate with $e^{-2x}\to Ae^{-2x}$ with any non -zero A, even 1.

Watch for $e^{-2x}\to Ae^{2x}$ which is M0 A0

A1: $\frac{dy}{dx}=-2e^{-2x}+2x$

M1: A correct method of finding the gradient of the normal at $x=0$

To score this the candidate must find the negative reciprocal of $\left.\frac{dy}{dx}\right|_{x=0}$

So for example candidates who find $\frac{dy}{dx}=e^{-2x}+2x$ should be using a gradient of -1

Candidates who write down $\frac{dy}{dx}=-2$ (from their calculators?) have an opportunity to score this mark and the next.

M1: An attempt at the equation of the normal at $(0,-2)$

To score this mark the candidate must be using the point $(0,-2)$ and a gradient that has been **changed from** $\left.\frac{dy}{dx}\right|_{x=0}$

Look for $y-(-2)=changed\left|\frac{dy}{dx}\right|_{x=0}(x-0)$ or $y=mx-2$ where $m=changed\left|\frac{dy}{dx}\right|_{x=0}$

If there is an attempt using $y=mx+c$ then it must proceed using $(0,-2)$ with $m=$ $changed\left|\frac{dy}{dx}\right|_{x=0}$

A1: $y=\frac{1}{2}x-2$ cso with as well as showing the correct differentiation.

So reaching $y=\frac{1}{2}x-2$ from $\frac{dy}{dx}=-2e^{2x}+2x$ is A0

If it is not simplified (or written in the required form) you may award this if $y=\frac{1}{2}x-2$ is seen in part (b)

(b)

M1: Equates $y=e^{-2x}+x^2-3$ and their $y=mx+c, m\neq 0$ and proceeds to $x^2=...$
Condone an attempt for this M mark where the candidate uses an adapted $y=mx+c$ in an attempt to get the printed answer.

A1*: Proceeds to $x=\sqrt{1+\frac{1}{2}x-e^{-2x}}$. It is a printed answer but you may accept a different order $x=\sqrt{1-e^{-2x}+\frac{1}{2}x}$

For this mark, the candidate must start with a normal equation of $y=\frac{1}{2}x-2$ oe found in (a). It can be awarded when the candidate finds the equation incorrectly, for example from $\frac{dy}{dx}=-2e^{2x}+2x$

(c)

M1: Sub $x_1=1$ in $x=\sqrt{1+\frac{1}{2}x-e^{-2x}}$ to find x_2. May be implied by $\sqrt{1+0.5-e^{-2}}$ oe or awrt 1.17

A1: $x_2=$ awrt 1.168, $x_3=$ awrt 1.220 3dp. Condone 1.22 for x_3

Mark these in the order given, the subscripts are not required and incorrect ones may be ignored.

Question Number	Scheme	Marks
5(a)	Either $k>13$ or $k=3$ Both $k>13$ $k=3$	B1 B1 **(2)**
(b)	Smaller solution: $2(5-x)+3=\frac{1}{2}x+10 \Rightarrow x=\frac{6}{5}$ Larger solution: $-2(5-x)+3=\frac{1}{2}x+10 \Rightarrow x=\frac{34}{3}$	M1 A1 M1 A1 **(4)**
(c)	$(6,12)$	B1B1 **(2)** **(8 marks)**

(a)

B1: Either $k>13$ or $k=3$ Condone $k \geqslant 13$ instead of $k>13$ for this mark only. Also condone $y \leftrightarrow k$

Do not accept $k \geq 3$ for B1

B1: Both $k>13$, $k=3$ with no other restrictions. Accept and / or / , between the two solutions

(b)

M1: An acceptable method of finding **the smaller intersection**. The initial equation must be of the correct form and it must lead to a value of *x*. For example $2(5-x)+3=\frac{1}{2}x+10 \Rightarrow x=...$ or $5-x=\left(\frac{1}{4}x+\frac{7}{2}\right)$

A1: For $x=\frac{6}{5}$ or equivalent such as 1.2 Ignore any reference to the y coordinate

M1: An acceptable method of finding **the larger intersection**. The initial equation must be of the correct form and it must lead to a value of *x*. For example $-2(5-x)+3=\frac{1}{2}x+10 \Rightarrow x=...$ or $5-x=-\left(\frac{1}{4}x+\frac{7}{2}\right)$

A1: For $x=\frac{34}{3}$ or equivalent such as $11.\dot{3}$ Ignore any reference to the y coordinate

If there are any extra solutions in addition to the correct two, then withhold the final A1 mark.

ISW if the candidate then refers back to the range in (a) and deletes a solution

....

Alt method by squaring

M1: $2|5-x|+3=\frac{1}{2}x+10 \Rightarrow 4(5-x)^2=\left(\frac{1}{2}x+7\right)^2$ oe. In the main scheme the equation must be correct of the correct form but in this case you may condone '2' not being squared

A1: Correct 3TQ. The $=0$ may be implied by subsequent work. $\frac{15}{4}x^2-47x+51=0$ oe

M1: Solves using an appropriate method $15x^2-188x+204=0 \Rightarrow (5x-6)(3x-34)=0 \Rightarrow x=..$

A1: Both $x=\frac{6}{5}$ $x=\frac{34}{3}$ and no others.

....

(c)

B1: Accept $p=6$ **or** $q=12$. Allow in coordinates as $x=6$ **or** $y=12$.

B1: For both $p=6$ **and** $q=12$. Allow in coordinates as $x=6$ **and** $y=12$

Allow embedded within a single coordinate $(6,12)$. So for example $(2,12)$ is scored B1 B0

Question Number	Scheme	Marks
6(i)	$\frac{\tan 2x+\tan 32°}{1-\tan 2x\tan 32°}=5\Rightarrow \tan(2x+32°)=5$ $\Rightarrow x=\frac{\arctan 5-32°}{2}$ $\Rightarrow x=\text{awrt }23.35°,-66.65°$	B1 M1 A1A1 **(4)**
(ii)(a)	$\tan(3\theta-45°)=\frac{\tan 3\theta-\tan 45°}{1+\tan 45°\tan 3\theta}=\frac{\tan 3\theta-1}{1+\tan 3\theta}$	M1A1* **(2)**
(b)	$(1+\tan 3\theta)\tan(\theta+28°)=\tan 3\theta-1$ $\Rightarrow \tan(\theta+28°)=\tan(3\theta-45°)$ $\theta+28°=3\theta-45°\Rightarrow \theta=36.5°$ $\theta+28°+180°=3\theta-45°\Rightarrow \theta=126.5°$	 B1 M1A1 dM1A1 **(5)** **(11 marks)**
6(i) ALT 1	$\frac{\tan 2x+\tan 32°}{1-\tan 2x\tan 32°}=5\Rightarrow \tan 2x=\frac{5-\tan 32°}{1+5\tan 32°}=awrt1.06$ $\Rightarrow x=\frac{\arctan\left(\frac{5-\tan 32°}{1+5\tan 32°}\right)}{2}$ $\Rightarrow x=23.35°,-66.65°$	B1 M1 A1A1 **(4)**
6(ii) ALT 2	$\frac{\tan 2x+\tan 32°}{1-\tan 2x\tan 32°}=5\Rightarrow \frac{2\tan x}{1-\tan^2 x}+\tan 32°=5-5\times\frac{2\tan x}{1-\tan^2 x}\tan 32°$ $\Rightarrow (5-\tan 32°)\tan^2 x+(2+10\tan 32°)\tan x+\tan 32°-5=0$ OR $\Rightarrow awrt\ 4.38\tan^2 x+8.25\tan x-4.38=0$ Quadratic formula $\Rightarrow \tan x=0.4316,-2.3169\Rightarrow x=..$ $\Rightarrow x=23.35°,-66.65°$	 B1 M1 A1 A1 **(4)**

(i)

B1: Stating or implying by subsequent work $\tan(2x+32°)=5$

M1: Scored for the correct order of operations from $\tan(2x\pm 32°)=5$ to $x=..$ $\quad x=\frac{\arctan 5\pm 32°}{2}$

This may be implied by one correct answer

A1: One of awrt $x=23.3/23.4°,-66.6/-66.7°$ **One dp accuracy required for this penultimate mark.**

A1: Both of $x=\text{awrt }23.35°,-66.65°$ and no other solutions in the range $-90°<x<90°$

Using Alt I

B1: $\tan 2x = \text{awrt}1.06$

M1: For attempting to make $\tan 2x$ the subject followed by correct inverse operations to find a value for x from their $\tan 2x = k$

If they write down $\tan(2x+32°) = 5$ and then the answers that is fine for all 4 marks.

Answers mixing degrees and radians can only score the first B1

(ii)(a)

M1: States or implies (just rhs) $\tan(3\theta - 45°) = \dfrac{\tan 3\theta \pm \tan 45°}{1 \pm \tan 45° \tan 3\theta}$

A1*: Complete proof with the lhs, the correct identity $\dfrac{\tan 3\theta - \tan 45°}{1+\tan 45° \tan 3\theta}$ and either stating that $\tan 45° = 1$ or substituting $\tan 45° = 1$ (which may only be seen on the numerator) and proceeding to given answer.

It is possible to work backwards here $\dfrac{\tan 3\theta - 1}{1+\tan 3\theta} = \dfrac{\tan 3\theta - \tan 45°}{1+\tan 45° \tan 3\theta} = \tan(3\theta - 45°)$ with M1 A1 scored at the end. Do not allow the final A1* if there are errors.

(ii)(b)

B1: Uses (ii)(a) to state or imply that $\tan(\theta + 28°) = \tan\left(3\theta - 45°\right)$

Allow this mark for $\left(1+\tan 3\theta\right)\tan(\theta+28°) = \left(1+\tan 3\theta\right)\tan\left(3\theta - 45°\right)$

M1: $\theta + 28° = 3\theta - 45° \Rightarrow \theta = ..$

We have seen two incorrect methods that should be given M0.

$\tan(\theta+28°) = \tan\left(3\theta-45°\right) \Rightarrow \tan\left(3\theta-45°\right) - \tan(\theta+28°) = 0 \Rightarrow \left(3\theta-45°\right) - (\theta+28°) = 0 \Rightarrow \theta = ...$

and $\tan 3\theta - \tan 45° = \tan\theta + \tan 28° \Rightarrow 3\theta - 45° = \theta + 28° \Rightarrow \theta = ...$

A1: $\theta = 36.5°$ oe such as $\dfrac{73}{2}$

dM1: A correct method of finding a 2nd solution $\theta + 28° + 180° = 3\theta - 45° \Rightarrow \theta = ..$ The previous M must have been awarded. The method may be implied by their $\theta_1 + 90°$ but only if the previous M was scored.

It is an incorrect method to substitute the acute angle into one side of $\tan(\theta+28°) = \tan\left(3\theta-45°\right)$

Eg. $\tan(36.5+28°) = \tan\left(3\theta-45°\right)$ and use trig to find another solution.

A1: $\theta = 36.5°, 126.5°$ oe and no other solutions in the range.

The questions states 'hence' so the minimum expected working is $\tan(\theta+28°) = \tan\left(3\theta-45°\right)$. Full marks can be awarded when this point is reached.

(ii) (b) Alternative solution using compound angles.

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(ii) (b) Alternative solution using compound angles.

From the B1 mark, $\tan(\theta+28°) = \tan\left(3\theta-45°\right)$ they proceed to

$$\frac{\sin(\theta+28°)}{\cos(\theta+28°)} = \frac{\sin\left(3\theta-45°\right)}{\cos\left(3\theta-45°\right)} \Rightarrow \sin\left((3\theta-45°)-(\theta+28°)\right) = 0$$ via the compound angle identity

So, M1 is gained for an attempt at one value for $\sin\left(2\theta - 73°\right) = 0$, condoning slips andA1 for $\theta = 36.5°$

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Question Number	Scheme	Marks
7.(a)	Applies $\dfrac{vu'-uv'}{v^2}$ to $y=\dfrac{\ln\left(x^2+1\right)}{x^2+1}$ with $u=\ln\left(x^2+1\right)$ and $v=x^2+1$	
	$\dfrac{dy}{dx}=\dfrac{\left(x^2+1\right)\times\dfrac{2x}{x^2+1}-2x\ln\left(x^2+1\right)}{\left(x^2+1\right)^2}$	M1 A1
	$\dfrac{dy}{dx}=\dfrac{2x-2x\ln\left(x^2+1\right)}{\left(x^2+1\right)^2}$	A1
		(3)
(b)	Sets $2x-2x\ln\left(x^2+1\right)=0$	M1
	$2x\left(1-\ln\left(x^2+1\right)\right)=0\Rightarrow x=\pm\sqrt{e-1},$	M1,A1
	Sub $x=\pm\sqrt{e-1},0$ into $f(x)=\dfrac{\ln\left(x^2+1\right)}{x^2+1}$	dM1
	Stationary points $\left(\sqrt{e-1},\dfrac{1}{e}\right),\left(-\sqrt{e-1},\dfrac{1}{e}\right),\underline{\underline{(0,0)}}$	A1 <u>B1</u>
		(6)
		(9 marks)

(a)

M1: Attempts the quotient or product rule to achieve an expression in the correct form

Using the quotient rule achieves an expression of the form $\dfrac{dy}{dx}=\dfrac{\left(x^2+1\right)\times\dfrac{\dots}{x^2+1}-2x\ln\left(x^2+1\right)}{\left(x^2+1\right)^2}$

or the form $\dfrac{dy}{dx}=\dfrac{\dots-2x\ln\left(x^2+1\right)}{\left(x^2+1\right)^2}$ where $\dots=A$ or Ax

or using the product rule achieves and an expression $\dfrac{dy}{dx}=\left(x^2+1\right)^{-1}\times\dfrac{\dots}{x^2+1}-2x\left(x^2+1\right)^{-2}\ln\left(x^2+1\right)$

You may condone the omission of brackets ....especially on the denominator

A1: A correct un-simplified expression for $\dfrac{dy}{dx}$

$$\frac{dy}{dx}=\frac{\left(x^2+1\right)\times\dfrac{2x}{x^2+1}-2x\ln\left(x^2+1\right)}{\left(x^2+1\right)^2}\quad\text{or}\quad\frac{dy}{dx}=\left(x^2+1\right)^{-1}\times\frac{2x}{x^2+1}-2x\left(x^2+1\right)^{-2}\ln\left(x^2+1\right)$$

A1: $\dfrac{dy}{dx}=\dfrac{2x-2x\ln\left(x^2+1\right)}{\left(x^2+1\right)^2}$ or exact simplified equivalent such as $\dfrac{dy}{dx}=\dfrac{2x\left(1-\ln\left(x^2+1\right)\right)}{\left(x^2+1\right)^2}$.

Condone $\frac{dy}{dx} = \frac{2x - \ln\left(x^2+1\right)2x}{\left(x^2+1\right)^2}$ which may be a little ambiguous. The lhs $\frac{dy}{dx} =$ does not need to be seen. You may assume from the demand in the question that is what they are finding.
ISW can be applied here.

(b)

M1: Sets the numerator of their $\frac{dy}{dx}$, which must contain at least two terms, equal to 0

M1: For solving an equation of the form $\ln(x^2+1) = k, \quad k > 0$ to get at least one non- zero value of x. Accept decimal answers. $x = awrt \pm 1.31$ The equation must be legitimately obtained from a numerator = 0

A1: Both $x = \pm\sqrt{e-1}$ scored from ± a correct numerator Condone $x = \pm\sqrt{e^1 - 1}$

dM1: Substitutes any of their non zero solutions to $\frac{dy}{dx} = 0$ into $f(x) = \frac{\ln\left(x^2+1\right)}{x^2+1}$ to find at least one 'y' value. It is dependent upon both previous M's

A1: Both $\left(\sqrt{e-1}, \frac{1}{e}\right), \left(-\sqrt{e-1}, \frac{1}{e}\right)$ oe or the equivalent with $x = ..., y = ...$ ln e must be simplified

Condone $\left(\sqrt{e^1-1}, \frac{1}{e^1}\right), \left(-\sqrt{e^1-1}, \frac{1}{e^1}\right)$ but the y coordinates must be simplified as shown.

Condone $\left(\pm\sqrt{e-1}, \frac{1}{e}\right)$ Withhold this mark if there are extra solutions to these apart from (0,0)

It can only be awarded from ± a correct numerator

B1: $(0,0)$ or the equivalent $x = 0, y = 0$

Notes:

(1) A candidate can "recover" and score all marks in (b) when they have an incorrect denominator in part (a) or a numerator the wrong way around in (a)

(2) A candidate who differentiates $\ln\left(x^2+1\right) \to \frac{1}{x^2+1}$ will probably only score (a) 100 (b) 100000

(3) A candidate who has $\frac{vu' + uv'}{v^2}$ cannot score anything more than (a) 000 (b)100001 as they would have $k < 0$

(4) A candidate who attempts the product rule to get $\frac{dy}{dx} = \left(x^2+1\right)^{-1} \times \frac{1}{x^2+1} - \left(x^2+1\right)^{-2} \ln\left(x^2+1\right) = \frac{1 - \ln\left(x^2+1\right)}{\left(x^2+1\right)^2}$

can score (a) 000 (b) 110100 even though they may obtain the correct non zero coordinates.

Question Number	Scheme	Marks
8(a)	$\frac{\mathrm{d}}{\mathrm{d}\theta}(\sec\theta)=\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta)^{-1}=-1\times(\cos\theta)^{-2}\times-\sin\theta$	M1
	$=\frac{1}{\cos\theta}\times\frac{\sin\theta}{\cos\theta}$	
	$=\sec\theta\tan\theta$	A1*
		(2)
(b)	$x=\mathrm{e}^{\sec y}\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}y}=\mathrm{e}^{\sec y}\times\sec y\tan y$ oe	M1A1
	$\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{\mathrm{e}^{\sec y}\times\sec y\tan y}$	M1
	Uses $1+\tan^2 y=\sec^2 y$ with $\sec y=\ln x\Rightarrow\tan y=\sqrt{(\ln x)^2-1}$	M1
	$\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{x\times\ln x\times\sqrt{(\ln x)^2-1}}=\frac{1}{x\sqrt{(\ln x)^4-(\ln x)^2}}$ oe	A1
		(5)
		(7 marks)
Alt (b)	$\ln x=\sec y\Rightarrow\frac{1}{x}\frac{\mathrm{d}x}{\mathrm{d}y}=\sec y\tan y$	**M1A1**
	$\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{x\times\sec y\tan y}$	**M1**
	Uses $1+\tan^2 y=\sec^2 y$ and $\sec y=\ln x\Rightarrow\tan y=\sqrt{(\ln x)^2-1}$	**M1**
	$\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{x\times\ln x\times\sqrt{(\ln x)^2-1}}=\frac{1}{x\sqrt{(\ln x)^4-(\ln x)^2}}$ oe	**A1**
		(5)

(a)

M1: Uses the chain rule to get $\pm 1\times(\cos\theta)^{-2}\times\sin\theta$

Alternatively uses the quotient rule to get $\dfrac{\cos\theta\times 0\pm 1\times\sin\theta}{\cos^2\theta}$ condoning the denominator as $\cos\theta^2$

When applying the quotient rule it is very difficult to see if the correct rule has been used. So only withhold this mark if an incorrect rule is quoted.

A1*: Completes proof with no errors (see below *) and shows line $\dfrac{1}{\cos\theta}\times\dfrac{\sin\theta}{\cos\theta}$, $\dfrac{\tan\theta}{\cos\theta}$ or $\dfrac{\sin\theta}{\cos\theta\times\cos\theta}$ before the given answer. The notation should be correct so do not allow if they start $y=\sec\theta\Rightarrow\dfrac{dy}{dx}=\sec\theta\tan\theta$

* You do not need to see $\dfrac{d}{d\theta}(\sec\theta)=...$ or $\dfrac{dy}{d\theta}$ anywhere in the solution

(b)

M1 Differentiates to get the rhs as $e^{\sec y}\times...$

A1 Completely correct differential inc the lhs $\dfrac{dx}{dy}=e^{\sec y}\times\sec y\tan y$

M1 Inverts **their** $\dfrac{dx}{dy}$ to get $\dfrac{dy}{dx}$.

The variable used **must be** consistent. Eg $\dfrac{dx}{dy}=e^{\sec y}\Rightarrow\dfrac{dy}{dx}=\dfrac{1}{e^{\sec x}}$ is M0

M1 For attempting to use $1+\tan^2 y=\sec^2 y$ with $\sec y=\ln x$

(You may condone $\ln x^2\to 2\ln x$ for the method mark)

It may be implied by $\tan y=\sqrt{\pm(\ln x)^2\pm 1}$ They must have a term in $\tan y$ to score this.

A valid alternative would be attempting to use $1+\cot^2 y=\mathrm{cosec}^2 y$ with $\mathrm{cosec}\,y=\dfrac{1}{\sqrt{1-\frac{1}{\ln^2 x}}}$ oe

A1 $\dfrac{dy}{dx}=\dfrac{1}{x\sqrt{(\ln x)^4-(\ln x)^2}}$ or exact equivalents such as $\dfrac{dy}{dx}=\dfrac{1}{x\sqrt{\ln^4 x-\ln^2 x}}$

Do not isw here. Withhold this mark if candidate then writes down $\dfrac{dy}{dx}=\dfrac{1}{x\sqrt{4(\ln x)-2(\ln x)}}$

Also watch for candidates who write $\dfrac{dy}{dx}=\dfrac{1}{x\sqrt{\ln x^4-\ln x^2}}$ which is incorrect (without the brackets)

Question Number	Scheme	Marks
9.(a)	$R=\sqrt{5}$	B1
	$\tan\alpha=2\Rightarrow\alpha=\text{awrt }1.107$	M1A1 (3)
(b)(i)	$'40+9R^2'=85$	M1A1
(ii)	$\theta=\frac{\pi}{2}+1.107\Rightarrow\theta=\text{awrt }2.68$	B1ft (3)
(c)(i)	6	B1
(ii)	$2\theta-'1.107'=3\pi\Rightarrow\theta=\text{awrt }5.27$	M1A1 (3) **(9 marks)**

(a)

B1: Accept $R=\sqrt{5}$ **Do not accept** $R=\pm\sqrt{5}$

M1: For sight of $\tan\alpha=\pm2,\ \tan\alpha=\pm\frac{1}{2}$. Condone $\sin\alpha=2,\cos\alpha=1\Rightarrow\tan\alpha=\frac{2}{1}$

If R is found first, accept $\sin\alpha=\pm\frac{2}{R},\ \cos\alpha=\pm\frac{1}{R}$

A1: $\alpha=\text{awrt }1.107$. The degrees equivalent $63.4°$ is A0.

(b)(i)

M1: Attempts $'40+9R^2'$ OR $'40+3R^2'$ using their R.

Can be scored for sight of the statement $'40+9R^2'$

It can be done via calculus. The M mark will probably be awarded when $\left("\alpha"-\frac{\pi}{2}\right)=-0.464$ is substituted into $M(\theta)$

A1: 85 exactly. Without any method this scores both marks. Do not accept awrt 85.

(b)(ii)

B1ft: For awrt 2.68 or $\left(\frac{\pi}{2}+"\alpha"\right)$ A simple way would be to add 1.57 to their α to 2dp

Accept awrt $153.4°$ for candidates who work in degrees. Follow through in degrees on $90°+'\alpha'$

(c)(i)

B1: 6

(c)(ii)

M1: Using $2\theta\pm'1.107'=n\pi$ where n is a positive integer leading to a value for θ

In degrees for $2\theta\pm\text{their}'63.43'=180n$ where n is a positive integer leading to a value for θ

Another alternative is to solve $\tan2\theta=2$ so score for $\frac{180n+\arctan2}{2}$ or $\frac{\pi n+\arctan2}{2}$

A1: $\theta=\text{awrt }5.27$ or if candidate works in degrees awrt $301.7°$

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